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Unsteady Flow Phenomena in Nozzles

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Nomenclature

a = speed of sound
A = nozzle cross-sectional area
L = nozzle length
M = Mach number
P = pressure
t = time
x = length
γ = ratio of specific heats
λ = $(x/A)(dA/dx)$

Subscripts

b = back or surrounding pressure
e = nozzle exit
0 = stagnation
$*$ = nozzle throat

Since the early 1950's, several investigations have been made of the unsteady or "start-up" flow phenomena in nozzles.[1-3] All of this work has been chiefly of an analytical nature with little quantitative experimental corroboration. To alleviate this situation, an analytical and experimental study of these phenomena was made at Lehigh University.[4]

The particular problem investigated involved a two-dimensional nozzle of hyperbolic contour attached to large reservoir initially containing a stagnant pressurized gas. Gas flow was initiated through the bursting of a membrane at the nozzle exit, the gas discharging directly into the surroundings. The nozzles are operated under the conditions shown in Table 1.

Excluding regions containing or affected by shock waves, the transient flow fields in the nozzles were considered to be the quasi-one dimensional, unsteady, adiabatic, homentropic flow of a compressible ideal gas ($\gamma = 1.4$). The continuity,

Table 1 Conditions for nozzle operation

Nozzle	$\frac{A_e}{A_*}$	$\frac{P_b}{P_0}$	Steady flow operating characteristics
Convergent	1.0	0.528	$M_e = 1.0$
Convergent-divergent	1.04676	0.38606	$M_e = 1.25$
Convergent-divergent	1.6875	≤ 0.278	$M_e = 2.0$
Convergent-divergent	1.6875	0.744	Stationary normal shock at $A/A_* = 1.3376$

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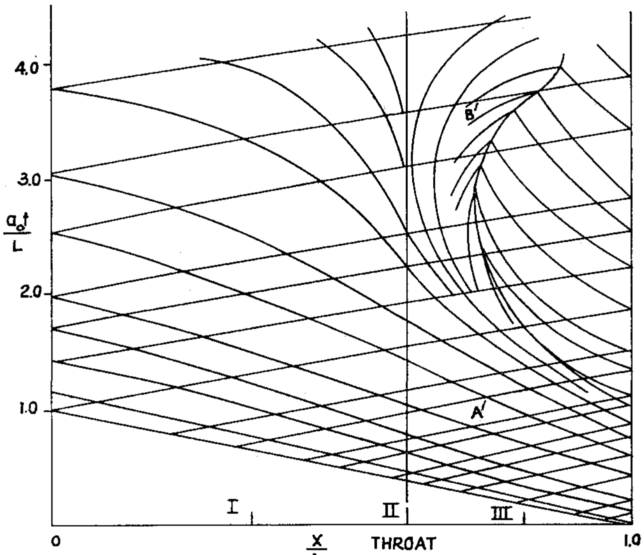


Fig. 1 Physical plane, Mach 2.0 nozzle ($P_b/P_0 = 0.744$).

momentum, and energy equations corresponding to these flow fields form a system of hyperbolic, quasi-linear, first-order partial differential equations. They were solved by the method of characteristics employing DeHaller's graphical-numerical technique.[5] The boundary conditions were: a) zero velocity and initial stagnation pressure at the nozzle inlet, b) pressure at the exit equal to that of the surroundings for subsonic outflow and determined by the characteristic net for supersonic outflow, c) flow-boundary contour was the physical-boundary contour, d) membrane burst generated a centered rarefaction wave. An example of the solutions is shown in Fig. 1 where the physical plane is sketched for the Mach 2.0 nozzle ($P_b/P_0 = 0.744$). From the physical and state planes, the theoretical pressure-time curves were determined at locations I, II, and III.

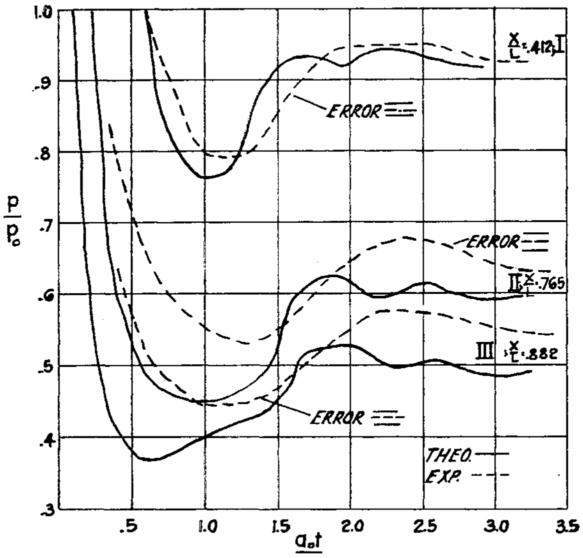
The experimental variation of pressure vs time was measured by Kistler piezoelectric crystal pressure transducers (rise time 6–7 μsec, natural frequency $\simeq$ 80,000 cps) with readout on a Tektronix Type 555 dual beam oscilloscope. The gas used was dry nitrogen (dew point $-73°$F). The data were reproducible to within the thickness of the scope trace.

Typical theoretical and experimental dimensionless pressure-time curves with corresponding error ranges are shown in Fig. 2. With the exception of the case involving the shock wave (and only for the latter stages of the transient at station III) the major qualitative trends predicted by theory were followed by the experimental curves, although the latter were quantitatively higher and lagged behind the former. The disagreement between the curves at station III for the shock-wave case was probably due to shock-wave bifurcation. Steady flow experiments[6] revealed that shock waves of the strength encountered for this case bifurcate completely forming a "pseudo" shock wave, which extends throughout the rest of the divergent section. This results in a higher pressure than predicted at station III.

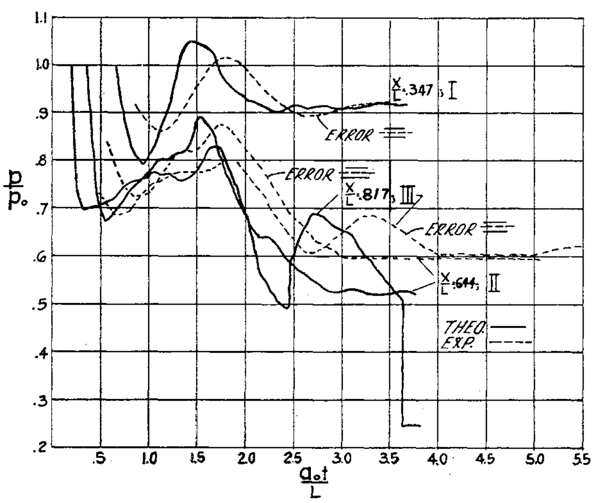
In order to determine the factors causing the quantitative discrepancies (too large to be due to experimental error), a re-evaluation was made of the initially postulated assumptions employed in the theory. This investigation revealed that discrepancies were due principally to the following incorrect assumptions:

1) Assumptions that the inital rarefaction wave was centered, the inlet boundary condition was zero velocity, and initial reservoir pressure was immediately adjacent to the inlet:

Previous investigators[7,8] have shown that the bursting of a membrane does not produce a centered rarefaction wave.



a) Mach 1.25 nozzle.



b) Mach 2.0 nozzle ($P_b/P_0 = 0.744$).

Fig. 2 Theoretical and experimental dimensionless pressure-time curves.

Tests were conducted to determine the form of the wave, and it was found to be affected by the nozzle contour, the operating pressure ratio, and membrane material characteristics.

The assumed inlet boundary condition is true only in the reservoir far from the inlet. In actuality a complex flow pattern exists at the nozzle inlet. A simplified model of the flow pattern was developed, based on potential flow theory, which enabled the continuation of the characteristic calculations into the reservoir proper until this boundary condition was approached.

Several of the characteristic diagrams were reconstructed employing the experimentally determined initial rarefaction wave and the modified inlet boundary condition. The Mach 1.25 nozzle results are shown in Fig. 3 where it is seen that agreement has been greatly improved during the first half of the transient period. During the second half, the experimental pressures again become higher and lag behind the theoretical. The calculations revealed that most of the lag is due to the nature of the membrane burst.

2) Assumption that the flow-boundary contour was the physical nozzle contour:

A steady flow analysis was made to determine the effect of the boundary-layer displacement thickness on the flow-

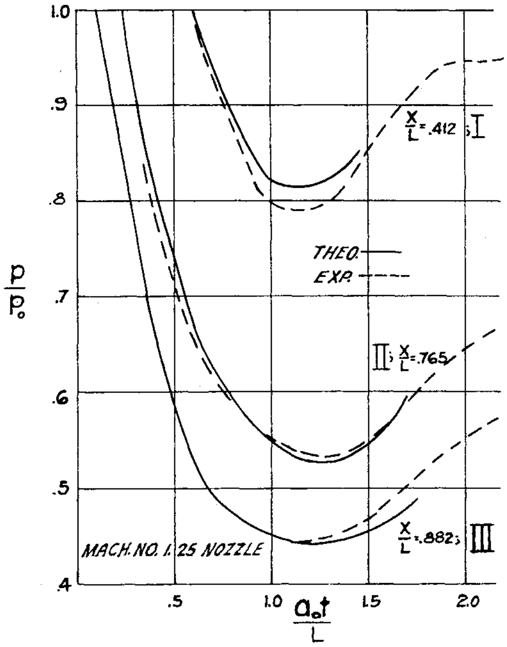


Fig. 3 Theoretical and experimental dimensionless pressure-time curves for Mach 1.25 nozzle employing actual initial rarefaction wave and the modified inlet boundary condition.

boundary contour employing a technique used by Simmons.[9] These calculations resulted in raising the theoretical pressures to within 4% of the experimental pressures as opposed to the 10% difference when the boundary layer was ignored. Since the boundary-layer growth has such a large effect on the final steady-state pressures, one might expect it has a large effect during the latter stages of the transient flow period as well, which partly explains the results of part 1.

3) Assumption that the flow is adiabatic and inviscid:

Resolving the characteristic equations at points A and B in Fig. 1, including heat-transfer and friction effects, resulted in a theoretical pressure increase of at most 3%.

All of the other assumptions were found to be valid to within calculational and experimental error for the contours investigated ($-1.0 \leq \lambda \leq 2.0$).

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